

MPI-PhT/94-43

TUM-T31-65/94

June 1994

Parity Violating Longitudinal Muon Polarization in $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ Beyond Leading Logarithms *

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Abstract

We generalize the existing analyses of the parity violating muon polarization asymmetry Δ_{LR} in $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ beyond the leading logarithmic approximation. The inclusion of next-to-leading QCD corrections reduces the residual dependence on the renormalization scales, which is quite pronounced in the leading order. This leads to a considerably improved accuracy in the perturbative calculation of the short distance dominated quantity Δ_{LR} . Accordingly this will also allow to obtain better constraints on the Wolfenstein parameter ϱ from future measurements of Δ_{LR} . For $-0.25 \leq \varrho \leq 0.25$, $V_{cb} = 0.040 \pm 0.004$ and $m_t = (170 \pm 20) \text{ GeV}$ we find $3.0 \cdot 10^{-3} \leq |\Delta_{LR}| \leq 9.6 \cdot 10^{-3}$.

*Supported by the German Bundesministerium für Forschung und Technologie under contract 06 TM 732 and by the CEC science project SC1-CT91-0729.

It has been pointed out by Savage and Wise [1] that measurements of muon polarization in $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay can give valuable information on the weak mixing angles and in particular on the parameter ϱ in the Wolfenstein parametrization [2]. Indeed as shown in [1, 3], the parity-violating asymmetry

$$|\Delta_{LR}| = \left| \frac{\Gamma_R - \Gamma_L}{\Gamma_R + \Gamma_L} \right| = r \cdot |\text{Re}\xi| \quad (1)$$

is dominated by the short distance contributions of Z-penguin and W-box diagrams with internal charm and top quark exchanges, while the total rate is completely determined by the one-photon exchange amplitude. The interference of this leading amplitude with the small short distance piece is the source of the asymmetry Δ_{LR} . Here Γ_R (Γ_L) is the rate to produce right- (left-) handed μ^+ , that is μ^+ with spin along (opposite to) its three-momentum direction. The factor r arises from phase space integrations. It depends only on the particle masses m_K , m_π and m_μ , on the form factors of the matrix element $\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle$, as well as on the form factor of the $K^+ \rightarrow \pi^+ \gamma^*$ transition, relevant for the one-photon amplitude. In addition r depends on a possible cut which may be imposed on θ , the angle between the three-momenta of the μ^- and the pion in the rest frame of the $\mu^+ \mu^-$ pair. Without any cuts one has $r = 2.3$ [3]. If $\cos \theta$ is restricted to lie in the region $-0.5 \leq \cos \theta \leq 1.0$, this factor is increased to $r = 4.1$. As discussed in [3], such a cut in $\cos \theta$ could be useful since it enhances Δ_{LR} by 80% with only a 22% decrease in the total number of events. $\text{Re}\xi$ is a purely short distance function depending only on CKM parameters, the QCD scale $\Lambda_{\overline{MS}}$ and the quark masses m_t and m_c . We will discuss it in detail below.

Δ_{LR} as given in (1) has also been considered by Bélanger et al. [4], who emphasized its close relation to the short distance part of the decay amplitude $K_L \rightarrow \mu^+ \mu^-$. Unfortunately the authors of ref. [4] did not include the internal charm contribution to Δ_{LR} . As we will show explicitly below the charm contribution cannot be neglected as its presence increases the extracted value of ϱ by roughly $\Delta\varrho = 0.2$.

Let us briefly summarize the theoretical situation of Δ_{LR} . The "kinematical" factor r can be essentially obtained from experimental input on the particle masses and the form factors. The form factor f describing the $K^+ \rightarrow \pi^+ \gamma^*$ vertex has been discussed in detail in [5] within chiral perturbation theory. While the

imaginary part can be reliably predicted, the real part is only determined up to a constant to be fitted from experiment. On the other hand, data on $K^+ \rightarrow \pi^+ e^+ e^-$ decay [6] allow to extract the absolute value of this form factor directly. We will adopt this approach, following [3]. Since the imaginary part of f is quite small [3, 5], we then also have the real part $\text{Re}f$. In principle $\text{Im}f$ could yield an extra contribution in (1) proportional to $\text{Im}\xi$. We have checked, based on the approach of [3] that this contribution is below 1% of the dominant part shown in (1) and can therefore be safely neglected. Clearly the factor r involves some uncertainty due to the experimental errors in the form factors, which can however be further reduced by future improved measurements. For the present discussion we will assume fixed numerical values for r .

Besides the short distance part of Δ_{LR} there are also potential long distance contributions coming from two-photon exchanges, which have also been discussed by the authors of [3]. These are difficult to calculate in a reliable manner, but the estimates given in [3] indicate that this contribution is substantially smaller than the short distance part, although it cannot be fully neglected. Therefore the short distance effects are expected to safely dominate the quantity Δ_{LR} and we shall concentrate our discussion on this part, keeping in mind the possible existence of non-negligible long distance corrections.

The short distance physics leading to Δ_{LR} is generally considered to be very clean, as it can be treated within a perturbative framework. However this does not mean that it is free of theoretical uncertainties. An indication of the involved error due to the necessary truncation of the perturbation series in the strong coupling constant α_s can be obtained by studying the sensitivity of a physical quantity to the relevant renormalization scales on which it should not depend in principle. The existing short distance calculations of Δ_{LR} [1, 3, 4] include QCD corrections in the leading logarithmic approximation (LLA) [7]. As it turns out, they suffer from sizable theoretical uncertainties due to the residual scale dependences. The main purpose of our letter is to extend the analyses of [1, 3, 4] beyond the leading logarithmic approximation thereby reducing considerably the theoretical uncertainties in question. To this end we will use our next-to-leading order analysis of $K_L \rightarrow \mu^+ \mu^-$ presented in [8, 9].

Our discussion of the Cabibbo-Kobayashi-Maskawa matrix will be based on the standard parametrization [10], which can equivalently be rewritten in terms

of the Wolfenstein parameters (λ , A , ϱ , η) through the definitions [11]

$$s_{12} \equiv \lambda \quad s_{23} \equiv A\lambda^2 \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\varrho - i\eta) \quad (2)$$

The unitarity structure of the CKM matrix is conventionally represented through the unitarity triangle in the complex plane with coordinates $(0, 0)$, $(1, 0)$ and $(\bar{\varrho}, \bar{\eta})$ where

$$\bar{\varrho} + i\bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \quad (3)$$

To better than 0.1% accuracy $\bar{\varrho} = \varrho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$.

Following [8, 9] it is straightforward to generalize the expression for $\text{Re}\xi$ of [3] beyond leading logarithms. We find

$$\text{Re}\xi = \kappa \cdot \left[\frac{\text{Re}\lambda_c}{\lambda} P_0 + \frac{\text{Re}\lambda_t}{\lambda^5} Y(x_t) \right] \quad (4)$$

$$\kappa = \frac{\lambda^4}{2\pi \sin^2 \Theta_W (1 - \frac{\lambda^2}{2})} = 1.66 \cdot 10^{-3} \quad (5)$$

Here $\lambda = |V_{us}| = 0.22$, $\sin^2 \Theta_W = 0.23$, $x_t = m_t^2/M_W^2$ and $\lambda_i = V_{is}^*V_{id}$. The function Y , relevant for the top contribution, is given by

$$Y(x) = Y_0(x) + \frac{\alpha_s}{4\pi} Y_1(x) \quad (6)$$

$$Y_0(x) = \frac{x}{8} \left[\frac{4-x}{1-x} + \frac{3x}{(1-x)^2} \ln x \right] \quad (7)$$

and

$$\begin{aligned} Y_1(x) = & \frac{4x + 16x^2 + 4x^3}{3(1-x)^2} - \frac{4x - 10x^2 - x^3 - x^4}{(1-x)^3} \ln x \\ & + \frac{2x - 14x^2 + x^3 - x^4}{2(1-x)^3} \ln^2 x + \frac{2x + x^3}{(1-x)^2} L_2(1-x) \\ & + 8x \frac{\partial Y_0(x)}{\partial x} \ln x_\mu \end{aligned} \quad (8)$$

where $x_\mu = \mu^2/M_W^2$ with $\mu = \mu_t = \mathcal{O}(m_t)$ and

$$L_2(1-x) = \int_1^x dt \frac{\ln t}{1-t} \quad (9)$$

The QCD correction Y_1 has been calculated in [8]. Next

$$P_0 = \frac{Y_{NL}}{\lambda^4} \quad (10)$$

where Y_{NL} represents the renormalization group expression for the charm contribution in next-to-leading logarithmic approximation (NLLA) calculated in [9]. It reads

$$Y_{NL} = C_{NL} - B_{NL}^{(-1/2)} \quad (11)$$

where C_{NL} is the Z-penguin part and $B_{NL}^{(-1/2)}$ is the box contribution, relevant for the case of final state leptons with weak isospin $T_3 = -1/2$. We have

$$\begin{aligned} C_{NL} = & \frac{x(m)}{32} K_c^{\frac{24}{25}} \left[\left(\frac{48}{7} K_+ + \frac{24}{11} K_- - \frac{696}{77} K_{33} \right) \left(\frac{4\pi}{\alpha_s(\mu)} + \frac{15212}{1875} (1 - K_c^{-1}) \right) \right. \\ & + \left(1 - \ln \frac{\mu^2}{m^2} \right) (16K_+ - 8K_-) - \frac{1176244}{13125} K_+ - \frac{2302}{6875} K_- + \frac{3529184}{48125} K_{33} \\ & \left. + K \left(\frac{56248}{4375} K_+ - \frac{81448}{6875} K_- + \frac{4563698}{144375} K_{33} \right) \right] \end{aligned} \quad (12)$$

where

$$K = \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \quad K_c = \frac{\alpha_s(\mu)}{\alpha_s(m)} \quad (13)$$

$$K_+ = K^{\frac{6}{25}} \quad K_- = K^{\frac{-12}{25}} \quad K_{33} = K^{\frac{-1}{25}} \quad (14)$$

$$\begin{aligned} B_{NL}^{(-1/2)} = & \frac{x(m)}{4} K_c^{\frac{24}{25}} \left[3(1 - K_2) \left(\frac{4\pi}{\alpha_s(\mu)} + \frac{15212}{1875} (1 - K_c^{-1}) \right) \right. \\ & \left. - \ln \frac{\mu^2}{m^2} - \frac{329}{12} + \frac{15212}{625} K_2 + \frac{30581}{7500} K K_2 \right] \end{aligned} \quad (15)$$

Here $K_2 = K^{-1/25}$, $m = m_c$, $x = m^2/M_W^2$. In (12) – (15) the two-loop expression has to be used for $\alpha_s(\mu)$ and $\mu = \mu_c = \mathcal{O}(m_c)$. The explicit μ -dependences in the next-to-leading order terms (8), (12) and (15) cancel the scale ambiguity of the leading contributions to the considered order in α_s . The consequences of this feature will be discussed later on. Numerical values of P_0 are given in table 1 where $m_c \equiv \bar{m}_c(m_c)$.

It is evident from (1) and (4) that, given $|\Delta_{LR}|$, one can extract $\text{Re}\lambda_t$:

$$\text{Re}\lambda_t = -\lambda^5 \frac{\frac{|\Delta_{LR}|}{r\kappa} - \left(1 - \frac{\lambda^2}{2}\right) P_0}{Y(x_t)} \quad (16)$$

Furthermore, using the standard parametrization of the CKM matrix we obtain from the definition (3)

$$\bar{\varrho} = \frac{\sqrt{1 + 4s_{12}c_{12}\text{Re}\lambda_t/s_{23}^2 - (2s_{12}c_{12}\text{Im}\lambda_t/s_{23}^2)^2} - 1 + 2s_{12}^2}{2c_{23}^2s_{12}^2} \quad (17)$$

	P_0		
$\Lambda_{\overline{MS}}, m_c [GeV]$	1.25	1.30	1.35
0.20	0.132	0.141	0.150
0.25	0.135	0.145	0.154
0.30	0.139	0.148	0.158
0.35	0.142	0.152	0.162

Table 1: The function P_0 for various $\Lambda_{\overline{MS}}$ and m_c .

Up to the very accurate approximations that $V_{cd}V_{cb}^*$ is real (error below 0.1%) and $c_{13} = 1$ (error less than 10^{-5}) (17) is an *exact* relation. Using the excellent approximation $\text{Im}\lambda_t = \eta A^2 \lambda^5$ [11], we see that a measured value of $\text{Re}\lambda_t$ determines by means of (17) a curve in the (ϱ, η) -plane. Since the dependence on $\text{Im}\lambda_t$ is however very small, this curve will be almost parallel to the η -axis. Thus, knowledge of $|\Delta_{LR}|$, hence $\text{Re}\lambda_t$, implies a value for ϱ (or $\bar{\varrho}$) almost independently of $\text{Im}\lambda_t$. For simplicity we shall neglect $\text{Im}\lambda_t$ in (17) completely, which introduces a change in $\bar{\varrho}$ of at most 0.01. It is evident that the more general treatment can be easily restored if desired. Note that the charm sector contributes to $\bar{\varrho}$ the non-negligible portion

$$\Delta\bar{\varrho}_{\text{charm}} \approx P_0/(A^2 Y(x_t)) \approx 0.2 \quad (18)$$

In order to demonstrate briefly the phenomenological consequences of the next-to-leading order calculation we consider the following scenario. We assume that the asymmetry Δ_{LR} is known to within $\pm 10\%$

$$\Delta_{LR} = (6.0 \pm 0.6) \cdot 10^{-3} \quad (19)$$

where a cut on $\cos\theta$, $-0.5 \leq \cos\theta \leq 1.0$, is understood. Next we take $(m_i \equiv \bar{m}_i(m_i))$

$$m_t = (170 \pm 5) GeV \quad m_c = (1.30 \pm 0.05) GeV \quad V_{cb} = 0.040 \pm 0.001 \quad (20)$$

$$\Lambda_{\overline{MS}} = (0.30 \pm 0.05) GeV \quad (21)$$

The errors quoted here seem quite reasonable if one keeps in mind that it will take at least ten years to achieve the accuracy assumed in (19). The value of m_t

		$\Delta(\Delta_{LR})$	$\Delta(m_t)$	$\Delta(V_{cb})$	$\Delta(m_c)$	$\Delta(\Lambda_{\overline{MS}})$
$\bar{\varrho}$	-0.06	± 0.13	± 0.05	± 0.06	± 0.01	± 0.00

Table 2: $\bar{\varrho}$ determined from Δ_{LR} for the scenario described in the text together with the uncertainties related to various input parameters.

in (20) is in the ball park of the most recent results of the CDF collaboration [12].

In table 2 we have displayed the central value for $\bar{\varrho}$ as it is extracted from Δ_{LR} ((16) and (17)) in our example, along with the uncertainties due to the parameters involved. This is intended to indicate the sensitivity of $\bar{\varrho}$ to the relevant input. The combined errors due to a simultaneous variation of several parameters may be obtained to a good approximation by simply adding the errors from table 2. It is interesting to compare these numbers with the renormalization scale ambiguities, which inevitably limit the accuracy of the short-distance calculation. If we vary the scale in the charm- and in the top sector as $1\text{GeV} \leq \mu_c \leq 3\text{GeV}$ and $100\text{GeV} \leq \mu_t \leq 300\text{GeV}$, respectively, keeping all other parameters at their central values, we obtain the following range for $\bar{\varrho}$

$$-0.15 \leq \bar{\varrho} \leq -0.03 \quad (\text{NLLA}) \quad (22)$$

$$-0.31 \leq \bar{\varrho} \leq 0.02 \quad (\text{LLA}) \quad (23)$$

We would like to emphasize the following points:

- The error in $\bar{\varrho}$ from (22), which illustrates the theoretical uncertainty of the short distance piece alone, is not negligible. It seems however moderate when compared to the errors shown in table 2. We stress that (22) is based on the complete next-to-leading order result for $\text{Re}\xi$. If only the leading log approximation is used instead, the range obtained for $\bar{\varrho}$ is by almost a factor of 3 larger (23).
- The error in (22) is almost entirely due to the charm sector. Indeed, if we vary only μ_c , keeping $\mu_t = m_t$ fixed, the corresponding interval for $\bar{\varrho}$ reads $(-0.14, -0.03)$. This illustrates once more, that the charm sector, being the dominant source of theoretical error in the short distance contribution to Δ_{LR} , should not be neglected.

- In the case $x \ll 1$, which is relevant for the charm contribution, the function Y has a very special structure. Expanding the renormalization group result Y_{NL} to first order in α_s one finds (here $x = m_c^2/M_W^2$)

$$Y_{NL} \doteq \frac{x}{2} + \frac{\alpha_s}{4\pi} x \ln^2 x \quad (24)$$

We observe that the leading logarithms $\sim x \ln x$, present in the Z-penguin- and the box part, have canceled in Y_{NL} , leaving the subleading term $x/2$ as the only contribution in the limit $\alpha_s = 0$. On the other hand QCD effects generate an $\alpha_s x \ln^2 x$ "correction", which is of the order $\mathcal{O}(x \ln x)$, hence a leading logarithmic term! As a first consequence the charm function Y is enhanced considerably (by a factor of ~ 2.5) through strong interaction corrections, compared to the non-QCD result. (This feature is in a sense similar to the case of the rare decay $b \rightarrow s\gamma$.) A second point is that the $x/2$ term, though formally subleading, is important numerically. Working within LLA one is then faced with the problem of how to deal with this term since it should strictly speaking be omitted in this approximation. Let us illustrate this issue in terms of the $\bar{\varrho}$ determination in our above example. We find $\bar{\varrho} = -0.12$ if we use the LLA formulae (with $\mu_c = m_c$, $\mu_t = m_t$) and simply add the $x/2$ piece. By contrast, omitting this term and using the strict leading log result we obtain $\bar{\varrho} = -0.20$. The scale ambiguities are very similar in both cases, roughly three times as big as in the next-to-leading order discussed above. For definiteness we have included the $x/2$ part to obtain (23).

The problem of the $x/2$ term is naturally removed in the next-to-leading logarithmic approximation (Y_{NL}) where this contribution is consistently taken into account.

Finally we give the standard model expectation for Δ_{LR} , based on the short distance contribution in (4), for the Wolfenstein parameter ϱ in the range $-0.25 \leq \varrho \leq 0.25$, $V_{cb} = 0.040 \pm 0.004$ and $m_t = (170 \pm 20) \text{ GeV}$. Including the uncertainties due to m_c , $\Lambda_{\overline{MS}}$, μ_c and μ_t and imposing the cut $-0.5 \leq \cos \theta \leq 1$, we find

$$3.0 \cdot 10^{-3} \leq |\Delta_{LR}| \leq 9.6 \cdot 10^{-3} \quad (25)$$

employing next-to-leading order formulae. Anticipating improvements in V_{cb} , m_t and ϱ we also consider a future scenario in which $\varrho = 0.00 \pm 0.02$, $V_{cb} =$

0.040 ± 0.001 and $m_t = (170 \pm 5) \text{ GeV}$. The very precise determination of ϱ used here should be achieved through measuring CP asymmetries in B decays in the LHC era [13]. Then (25) reduces to

$$4.8 \cdot 10^{-3} \leq |\Delta_{LR}| \leq 6.6 \cdot 10^{-3} \quad (26)$$

In both of the scenarios the lower (upper) limit for Δ_{LR} would be smaller by $0.6 \cdot 10^{-3}$ ($1.3 \cdot 10^{-3}$) if the charm contribution was omitted.

In this letter we have generalized the short distance calculation of the muon polarization asymmetry Δ_{LR} in the decay $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ to next-to-leading order in QCD. We furthermore discussed the theoretical uncertainties involved in this analysis. We have demonstrated that the complete next-to-leading order calculation achieves a reduction of the rather large scale ambiguities in leading order by a factor of ~ 3 and is necessary to provide a satisfactory treatment of Δ_{LR} . This is particularly important since long distance contributions to Δ_{LR} seem to be small, though perhaps not fully negligible [3].

In any case a measurement of Δ_{LR} would yield a very interesting and useful piece of information on short distance flavordynamics and the unitarity triangle which is worth pursuing in future experiments.

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